PART - I (CHEMISTRY)

SECTION–I : (Total Marks : 24)
(Single Correct Answer Type)

This section contains **8 multiple choice questions**. Each question has four choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

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1. Oxidation states of the metal in the minerals haematite and magnetite, respectively, are

(A) II, III in haematite and III in magnetite

(B) II, III in haematite and II in magnetite

(C) II in haematite and II, III in magnetite

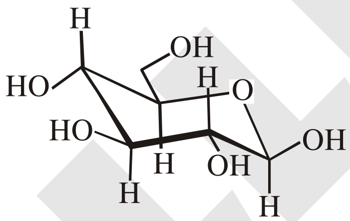
(D) III in haematite and II, III in magnetite

1. Ans. (D)

Haematite Fe_2O_3 — oxidation State of Fe = III

Magnetite Fe_3O_4 $(FeO.\ Fe_2O_3)$ — oxidation State of Fe = II, III

2. The following carbohydrate is



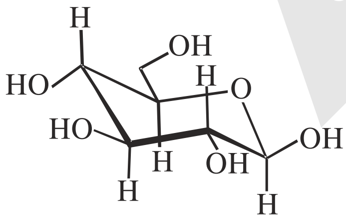
(A) a ketohexose

(B) an aldohexose

(C) an α-furanose

(D) an α-pyranose

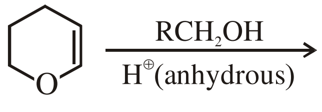
2. Ans. (B)



β-pyranose

(Aldohexose)

3. The major product of the following reaction is



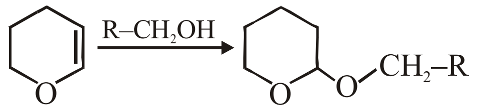
(A) a hemiacetal

(B) an acetal

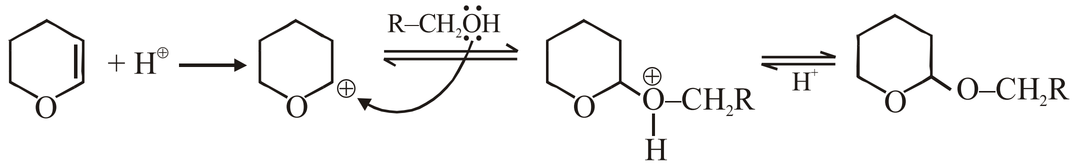
(C) an ether

(D) an ester

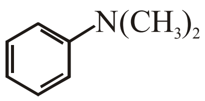
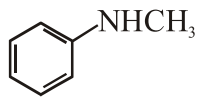
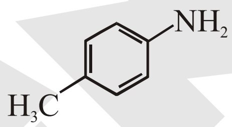
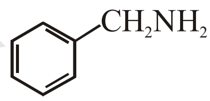
3. Ans.(B)



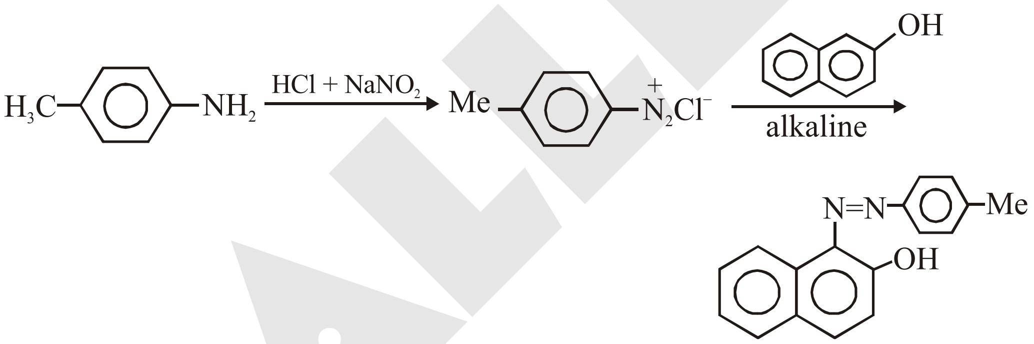
(An acetal)



4. Amongst the compounds given, the one that would form a brilliant coloured dye on treatment with $NaNO_2$ in dil. HCl followed by addition to an alkaline solution of β-naphthol is -

(A)  (B)  (C)  (D) 

4. Ans. (C)



Orange coloured dye

5. The freezing point (in °C) of a solution containing 0.1 g of $K_3[Fe(CN)_6]$ (Mol. Wt. 329) in 100 g of water (K_f = 1.86 K kg mol^{-1}) is -

(A) -2.3×10^{-2} (B) -5.7×10^{-2} (C) -5.7×10^{-3} (D) -1.2×10^{-2}

5. Ans. (A)

$\Delta T_f = m \times K_f \times i$

$$= \frac{0.1/329}{100} \times 1000 \times 1.86 \times 4$$

$= 0.02261$

$= 2.3 \times 10^{-2}$

$T_f = 0 - \Delta T_f = -2.3 \times 10^{-2}\,°C$

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6. Consider the following cell reaction :

$2Fe_{(s)}+O_{2(g)}+4H^+_{(aq)} \rightarrow 2Fe^{2+}_{(aq.)}+2H_2O(\ell)$ $\quad E° = 1.67$ V

At$[Fe^{2+}] = 10^{-3}$ M, $P(O_2) = 0.1$ atm and pH = 3, the cell potential at 25°C is -

(A) 1.47 V (B) 1.77 V (C) 1.87 V (D) 1.57 V

6. Ans. (D)

$2Fe_{(s)}+O_{2(g)}+4H^+_{(aq)} \longrightarrow 2Fe^{2+}_{(aq.)}+2H_2O(\ell)$

$$E = E° - \frac{0.06}{n}\log\frac{[Fe^{+2}]}{P_{O_2}.[H^+]^4}$$

$$= 1.67 - \frac{0.06}{4}\log\frac{(10^{-3})^2}{0.1\times(10^{-3})^4}$$

$$E = 1.67 - \frac{0.06}{4}\log 10^7$$

$$= 1.67 - \frac{0.06}{4}\times 7$$

$= 1.67 - 0.105$

$= 1.565$ V

$\simeq 1.57$ V

7. Passing H_2S gas into a mixture of Mn^{2+}, Ni^{2+}, Cu^{2+} and Hg^{2+} ions in an acidified aqueous solution precipitates

(A) CuS and HgS (B) MnS and CuS

(C) MnS and NiS (D) NiS and HgS

7. Ans. (A)

CuS and HgS will be precipitated when H_2S in passed through aqueous solution containing Mn^{2+}, Ni^{2+}, Cu^{2+} and Hg^{2+} ions in acidic medium.

8. Among the following complexes (**K–P**)

$K_3[Fe(CN)_6]$ **(K)**, $[Co(NH_3)_6]Cl_3$ **(L)**, $Na_3[Co(oxalate)_3]$ **(M)**, $[Ni(H_2O)_6]Cl_2$ **(N),**

$K_2[Pt(CN)_4]$ **(O)** and $[Zn(H_2O)_6]\,(NO_3)_2$ **(P)**

The diamagnetic complex are -

(A) K, L, M, N (B) K, M, O, P

(C) L, M, O, P (D) L, M, N, O

8. Ans. (C)

Diamanetic complexes are

(L) – $[Co(NH_3)_6]Cl_3$ **;** **(M) –** $Na_3[Co(oxalate)_3]$

(O) – $K_2[Pt(CN)_4]$ **;** **(P) –** $[Zn(H_2O)_6]\,(NO_3)_2$

SECTION–II : (Total Marks : 16)

(Multiple Correct Answer Type)

This section contains **4 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE** may be correct.

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9. Reduction of the metal centre in aqueous permanganate ion involves -

(A) 3 electrons in neutral medium (B) 5 electrons in neutral medium

(C) 3 electrons in alkaline medium (D) 5 electrons in acidic medium

9. Ans. (A,C,D)

$\overset{7+}{MnO_4^-} \xrightarrow{\text{acidic}} Mn^{2+}$

$\overset{7+}{MnO_4^-} \xrightarrow{\text{strong alkaline}} Mn^{6+}$

$\overset{7+}{MnO_4^-} \xrightarrow[\text{or neutral}]{\text{Faintly alkaline}} Mn^{4+}$

10. For the first order reaction

$$2N_2O_5(g) \longrightarrow 4NO_2(g) + O_2(g)$$

(A) the concentration of the reactant decreases exponentially with time

(B) the half-life of the reaction decreases with increasing temperature.

(C) the half-life of the reaction depends on the initial concentration of the reactant.

(D) the reaction proceeds to 99.6% completion in eight half-life duration.

10. Ans. (A,B,D)

$A_t = A_0\, e^{-kt}$

for option (D) $\frac{1}{t_{1/2}} \ln\frac{100}{50} = \frac{1}{t_{99.6\%}} \ln\frac{100}{0.4}$

$\frac{t_{99.6}}{t_{1/2}} = \frac{\ln 250}{\ln 2} = 7.965 \simeq 8$

11. The equilibrium

$$2Cu^{I} \rightleftharpoons Cu^{o} + Cu^{II}$$

in aqueous medium at 25°C shifts towards the left in the presence of

(A) NO_3^- (B) Cl^- (C) SCN^- (D) CN^-

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11. Ans. (B,C,D)

The eq^{n}. $2Cu^{I} \rightleftharpoons Cu^{o} + Cu^{II}$

shifts towards left in the presence of Cl^-, SCN^-, and CN^- due to formation of CuCl(ppt).
$[Cu(SCN)_4]^{-3}$ and $[Cu(CN)_4]^{-3}$ ions respectively.

12. The correct functional group X and the reagent/reaction conditions Y in the following scheme are

$$X—(CH_2)_4—X \xrightarrow[\text{(ii) } HO(O=)C-(CH_2)_4-C(=O)OH]{\text{(i) Y}} \text{condensation polymer}$$

(A) X = $COOCH_3$, Y = H_2/Ni/heat

(B) X = $CONH_2$, Y = H_2/Ni/heat

(C) X = $CONH_2$, Y = Br_2/NaOH

(D) X = CN, Y = H_2/Ni/heat

12 Ans.(A,B,C,D)

$$X–(CH_2)_4–X \xrightarrow[\text{(ii) } HOOC–(CH_2)_4–COOH]{\text{(i) y}} \text{Condensation polymer}$$

$$H_3COOC–(CH_2)_4–COOCH_3 \xrightarrow[\Delta]{H_2/Ni} \underset{\text{Hexane-1,6-diol}}{HO–(CH_2)_6–OH} \xrightarrow[\Delta]{HOOC–(CH_2)_4–COOH} \text{Condensation polyester}$$

$$H_2NOC–(CH_2)_4–CONH_2 \xrightarrow[\Delta]{H_2/Ni} \underset{\text{Hexamethylene diamine}}{H_2N–(CH_2)_6–NH_2} \xrightarrow[\Delta]{HOOC–(CH_2)_4–COOH} \text{Condensation polyamide}$$

$$H_2NOC–(CH_2)_4–CONH_2 \xrightarrow{Br_2/NaOH} \underset{\text{Tetramethylene diamine}}{H_2N–(CH_2)_4–NH_2} \xrightarrow[\Delta]{HOOC–(CH_2)_4–COOH} \text{Condensation polyamide}$$

$$NC–(CH_2)_4–CN \xrightarrow[\Delta]{H_2/Ni} \underset{\text{Hexamethylene diamine}}{H_2N–(CH_2)_6–NH_2} \xrightarrow[\Delta]{HOOC–(CH_2)_4–COOH} \text{Condensation polyamide}$$

SECTION–III : (Total Marks : 24)

(Integer Answer Type)

This Section contains **6 questions**. The answer to each of the questions is a **single-digit integer,** ranging from 0 to 9. The bubble corresponding to the correct answer is to be darkened in the ORS.

13. In 1 L saturated solution of AgCl [K_{sp}(AgCl) = 1.6×10^{-10}], 0.1 mol of CuCl [K_{sp}(CuCl) = 1.0×10^{-6}] is added. The resultant concentration of Ag^+ in the solution is 1.6×10^{-x}. The value of 'x' is.

13. Ans.(7)

$$AgCl \rightleftharpoons \underset{a}{Ag^+} + \underset{a+b}{Cl^-}$$

$$CuCl \rightleftharpoons \underset{b}{Cu^+} + \underset{(b+a)}{Cl^-}$$

$$\frac{K_{sp}(AgCl)}{K_{sp}(CuCl)} = \frac{a(a+b)}{b(b+a)} = \frac{a}{b} = \frac{[Ag^+]}{[Cu^+]} = \frac{1.6\times10^{-10}}{10^{-6}} = 1.6\times 10^{-4}$$

Since a<<< b

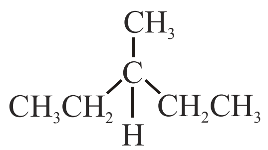
$\because \quad b^2 = 10^{-6} \Rightarrow b = 10^{-3}$

and $\quad a \times b = 1.6 \times 10^{-10}$

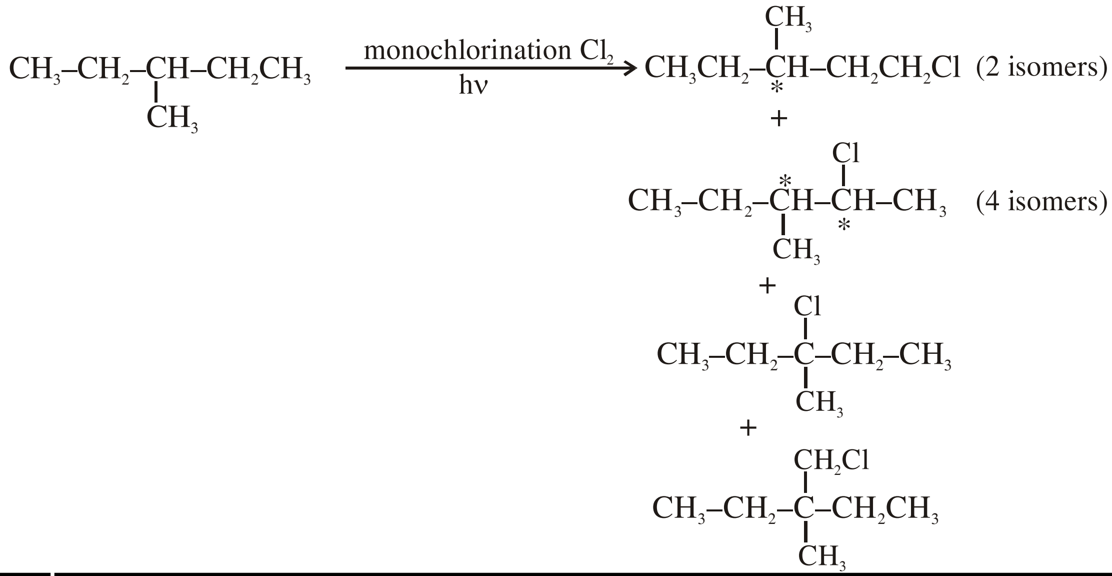
$\quad a = 1.6 \times 10^{-7} = 1.6 \times 10^{-x}$

so $\quad x = 7$

14. The maximum number of isomers (including stereoisomers) that are possible on mono-chlorination of the following compounds, is



14. Ans.(8)



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15. Among the following, the number of compounds that can react with PCl_5 to give $POCl_3$ is

O_2, CO_2, SO_2, H_2O, H_2SO_4 , P_4O_{10}

15. Ans. (4)

SO_2, H_2O, H_2SO_4, P_4O_{10}

$PCl_5 + H_2O \longrightarrow POCl_3 + 2\,HCl$

$6PCl_5 + P_4O_{10} \longrightarrow 10POCl_3$

$H_2SO_4 + PCl_5 \longrightarrow POCl_3 + SO_2Cl_2 + H_2O$

$SO_2 + PCl_5 \longrightarrow POCl_3 + SOCl_2$

16. The number of hexagonal faces that present in a truncated octahedron is.

16. Ans. (8)

In a truncated octahedron, there are 14 faces (8 regular hexagonal and 6 square), 36 edges and 24 vertices.

17. The volume (in mL) of 0.1M $AgNO_3$ required for complete precipitation of chloride ions present in 30 mL of 0.01M solution of $[Cr(H_2O)_5Cl]Cl_2$, as silver chloride is close to.

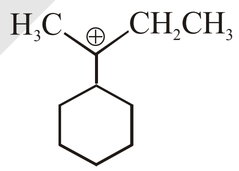
17. Ans.(6)

millimoles of $AgNO_3$ = 2 × millimoles of $(Cr(H_2O)_5Cl)Cl_2$

$0.1 \times V = 30 \times 2 \times 0.01 = 0.6$

V = 6 mL

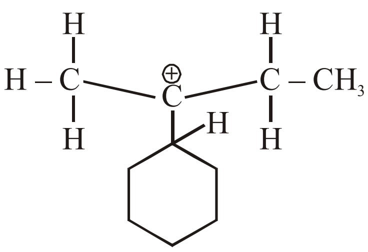
18. The total number of contributing structures showing hyperconjugation (involving C–H bonds) for the following carbocation is.



18. Ans. (6)

The contributing hyperconjugating structures (involving C – H bonds) are = 6

No. of α – H = 6



SECTION–IV : (Total Marks : 16)

(Matrix-Match Type)

This Section contains **2 questions**. Each question has **four statements** (A, B, C and D) given in **Column I** and five statements (p, q, r, s and t) in **Column II**. Any given statement in Column I can have correct matching with **ONE** or **MORE** statement(s) given in Column II. For example, if for a given question, statement B matches with the statements given in q and r, then for the particular question, against statement B, darken the bubbles corresponding to q and r in the ORS.

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19. Match the transformations in **Column-I** with appropriate option in **Column-II**

	Column-I		**Column-II**
(A)	$CO_2(s) \rightarrow CO_2(g)$	(p)	phase transition
(B)	$CaCO_3(g) \rightarrow CaO(s) + CO_2(g)$	(q)	allotropic change
(C)	$2H\bullet \rightarrow H_2(g)$	(r)	ΔH is positive
(D)	$P_{(white,\ solid)} \rightarrow P_{(red,solid)}$	(s)	ΔS is positive
		(t)	ΔS is negative

19. Ans. (A)→(p, r, s) ; (B)→(r, s) ; (C)→(t) ; (D)→(q, t)

$CO_2(s) \longrightarrow CO_{2(g)}$

phase transition

$\Delta H > 0,\ \Delta S > 0$

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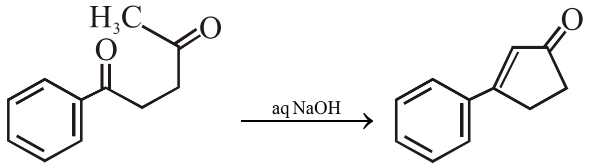
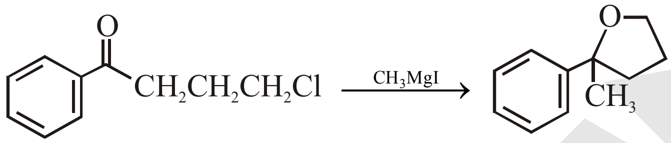
20. Match the reactions in **Column-I** with appropriate types of step/reactive intermediate involved in these reactions as given in **Column-II**

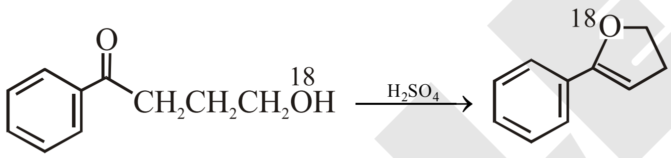
Column-I | **Column-II**

(A) 

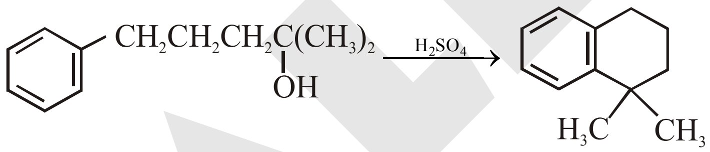
(p) Nucleophilic substitution

(B) 

(q) Electrophilic substitution

(C) 

(r) Dehydration

(D) 

(s) Nucleophilic addition

(t) Carbanion

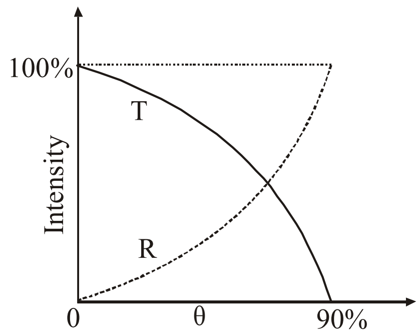
20. Ans. (A)→(r,s,t) ; (B)→(p, s, t) ; (C)→(r, s) ; (D)→(q, r)

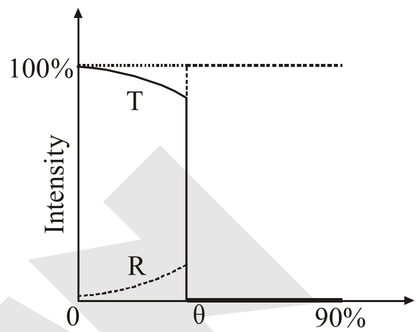
PART -II (PHYSICS)

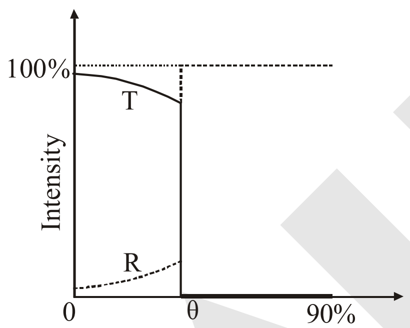
SECTION–I : (Total Marks : 24)
(Single Correct Answer Type)

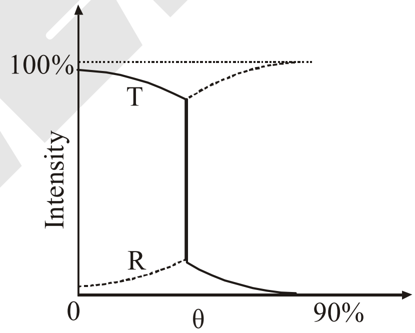
This section contains **8 multiple choice questions**. Each question has four choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

21. A light ray traveling in glass medium is incident on glass-air interface at an angle of incidence θ. The reflected (R) and transmitted (T) intensities, both as function of θ, are plotted. The correct sketch is

(A)


(B)


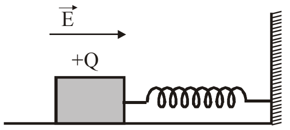
(C)


(D)


Ans. (C)

When θ =0°, maximum light is transmitted. At $\theta > \theta_C$ (critical angle), no further light is transmitted

22. A wooden block performs SHM on a frictionless surface with frequency, ν_0. The block carries a charge +Q on its surface. If now a uniform electric field $\vec{E}$ is switched-on as shown, then the SHM of the block will be



(A) of the same frequency and with shifted mean position
(B) of the same frequency and with the same mean position
(C) of changed frequency and with shifted mean position
(D) of changed frequency and with the same mean position

Ans. (A)

Time period of spring block system depends on spring constant and mass of block.
On applying electric field only the equilibrium position gets shifted.

23. The density of a solid ball is to be determined in an experiment. The diameter of the ball is measured with a screw gauge, whose pitch is 0.5 mm and there are 50 divisions on the circular scale. The reading on the main scale is 2.5 mm and that on the circular scale is 20 divisions. If the measured mass of the ball has a relative error of 2%, the relative percentage error in the density is

(A) 0.9% (B) 2.4% (C) 3.1% (D) 4.2%

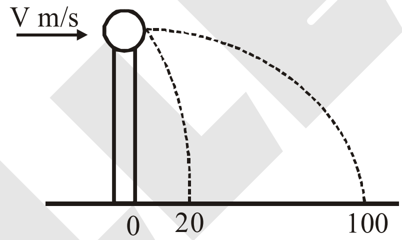
Ans. (C)

$$P = \frac{M}{\frac{4}{3}\pi r^3}, \; 100 \times \frac{\Delta P}{P} = \left(\frac{\Delta M}{M} + \frac{3\Delta r}{r}\right) \times 100$$

Δr = least count = 0.01 $\Rightarrow$ r = 2.72

$$\frac{\Delta P}{P} \times 100 = 2\% + \left(3 \times \frac{0.01}{2070}\right) \times 100 = 3.1\%$$

24. A ball of mass 0.2 kg rests on a vertical post of height 5m. A bullet of mass 0.01 kg, traveling with a velocity V m/s in a horizontal direction, hits the centre of the ball. After the collision, the ball and bullet travel independently. The ball hits the ground at a distance of 20 m and the bullet at a distance of 100 m from the foot of the post. The initial velocity V of the bullet is



(A) 250 m/s (B) $250\sqrt{2}$ m/s (C) 400 m/s (D) 500 m/s

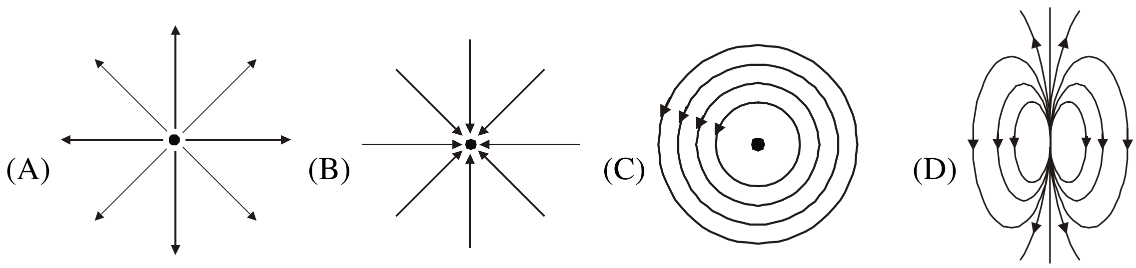
Ans. (D)

0.01 V = 0.2 u + 0.01 × 5 u

Time of flight t = 1 s; Range for ball = u × t ⇒ 20 = u × 1 ⇒ u = 20 m/s

⇒ V = 500 m/s

25. Which of the field patterns given below is valid for electric field as well as for magnetic field?



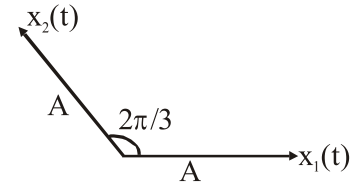
Ans. (C)

Magnetic field lines and induced electric field lines always from closed loop.

26. A point mass is subjected to two simultaneous sinusoidal displacements in x-direction, $x_1(t) = A\sin\omega t$ and $x_2(t) = A\sin\left(\omega t + \frac{2\pi}{3}\right)$. Adding a third sinusoidal displacement $x_3(t) = B\sin(\omega t + \phi)$ brings the mass to a complete rest. The values of B and ϕ are

(A) $\sqrt{2}A, \frac{3\pi}{4}$ (B) $A, \frac{4\pi}{3}$ (C) $\sqrt{3}A, \frac{5\pi}{6}$ (D) $A, \frac{\pi}{3}$

Ans. (B)

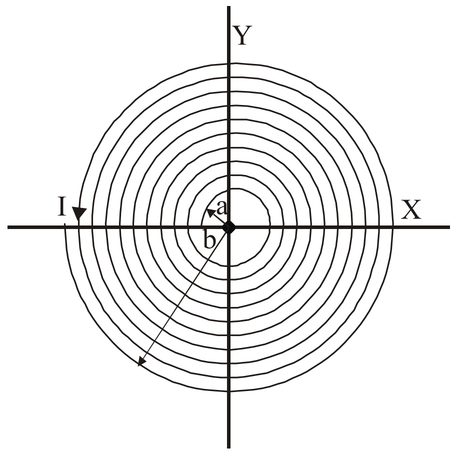


$x_1(t) + x_2(t) + x_3(t) = 0$

$x_3(t)$ has to be such that resultant is zero.

So it should make $\frac{4\pi}{3}$ from $x_1(t)$ anticlockwise.

27. A long insulated copper wire is closely wound as a spiral of 'N' turns. The spiral has inner radius 'a' and outer radius 'b'. The spiral lies in the X-Y plane and a steady current 'I' flows through the wire. The Z-component of the magnetic field at the center of the spiral is



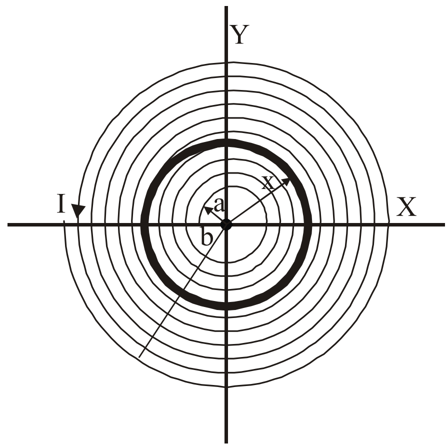
(A) $\frac{\mu_0 NI}{2(b-a)}\ell n\left(\frac{b}{a}\right)$

(B) $\frac{\mu_0 NI}{2(b-a)}\ell n\left(\frac{b+a}{b-a}\right)$

(C) $\frac{\mu_0 NI}{2b}\ell n\left(\frac{b}{a}\right)$

(D) $\frac{\mu_0 NI}{2b}\ell n\left(\frac{b+a}{b-a}\right)$

Ans. (A)



Taking an elemental strip of radius x and width dx.

Area of strip = $2\pi x\, dx$

Number of turns through area = $\frac{N}{b-a}dx$

$$\int dB = \int_a^b \frac{\mu_0 \frac{N}{(b-a)} I dx}{2x} = \frac{\mu_0 NI \ell n\left(\frac{b}{a}\right)}{2(b-a)}$$

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28. A satellite is moving with a constant speed 'V' in a circular orbit about the earth. An object of mass 'm' is ejected from the satellite such that it just escapes from the gravitational pull of the earth. At the time of its ejection, the kinetic energy of the object is

(A) $\frac{1}{2}mV^2$ (B) mV^2 (C) $\frac{3}{2}mV^2$ (D) $2mV^2$

Ans. (B)

KE of object = $\frac{1}{2}mv^2$ when it moves with satellite ; PE of object = $-mv^2$

At the time of ejection KE + PE = 0 to make it escape from gravitational pull.
KE = mv^2.

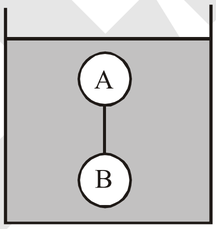
SECTION–II : (Total Marks : 16)

(Multiple Correct Answer Type)

This section contains **4 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE** may be correct.

---

29. Two solid spheres A and B of equal volumes but of different densities d_A and d_B are connected by a string. They are fully immersed in a fluid of density d_F. They get arranged into an equilibrium state as shown in the figure with a tension in the string. The arrangement is possible only if



(A) $d_A < d_F$ (B) $d_B > d_F$ (C) $d_A > d_F$ (D) $d_A + d_B = 2d_F$

Ans. (ABD)

$F_{Buoyant} = (m_a + m_B)g$; $2vd_Fg = v(d_A+d_B)g$
$d_A + d_B = 2d_F$. Therefore a, b, d

30. Which of the following statement(s) is/are correct?

(A) If the electric field due to a point charge varies as $r^{-2.5}$ instead of r^{-2}, then the Gauss law will still be valid.

(B) The Gauss law can be used to calculate the field distribution around an electric dipole.

(C) If the electric field between two point charges is zero somewhere, then the sign of the two charges is the same.

(D) The work done by the external force in moving a unit positive charge from point A at potential V_A to point B at potential V_B is $(V_B - V_A)$

Ans. (CD)

The field distribution for a dipole can not be calculated by using Gauss law only, therefore (C,D)

31. A series R-C circuit is connected to AC voltage source. Consider two cases ; (A) when C is without a dielectric medium and (B) when C is filled with dielectric of constant 4. The current I_R through the resistor and voltage V_C across the capacitor are compared in the two cases. Which of the following is/are true?

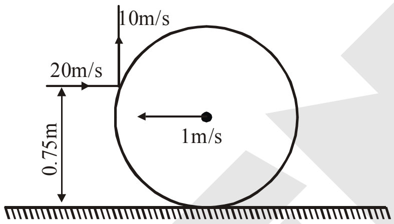
(A) $I_R^A > I_R^B$ (B) $I_R^A < I_R^B$ (C) $V_C^A > V_C^B$ (D) $V_C^A < V_C^B$

Ans. (BC)

X_C decreases therefore impedence decreases and current increases. $I_B > I_A$

As I_B increases the voltage across 'R' increases therefore V_C decreases.

32. A thin ring of mass 2 kg and radius 0.5 m is rolling without slipping on a horizontal plane with velocity 1 m/s. A small ball of mass 0.1 kg, moving with velocity 20 m/s in the opposite direction, hits the ring at a height of 0.75 m and goes vertically up with velocity 10 m/s. Immediately after the collision



(A) the ring has pure rotation about its stationary CM
(B) the ring comes to a complete stop
(C) friction between the ring and the ground is to the left
(D) there is no friction between the ring and the ground

Ans. (AC)

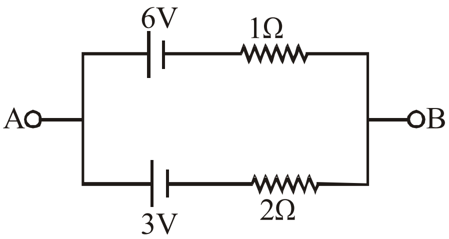
Since momentum of ball and ring has same magnitude but they are opposite in direction and final momentum of ball after the collision in horizontal direction is zero, therefore the ring has pure rotation about its stationary CM just after collision (assuming non–impulsive friction).

SECTION–III : (Total Marks : 24)

(Integer Answer Type)

This Section contains **6 questions**. The answer to each of the questions is a **single-digit integer,** ranging from 0 to 9. The bubble corresponding to the correct answer is to be darkened in the ORS.

33. Two batteries of different emfs and different internal resistances are connected as shown. The voltage across AB in volts is



Ans.(5)

$$v_{AB} = \frac{\frac{6}{1}+\frac{3}{2}}{\frac{1}{1}+\frac{1}{2}} = \frac{\frac{15}{2}}{\frac{3}{2}} = 5$$

34. A series R-C combination is connected to an AC voltage of angular frequency ω=500 radian/s. If the impedance of the R-C circuit is $R\sqrt{1.25}$, the time constant (in millisecond) of the circuit is

Ans. (4)

$$1.25\,R^2 = R^2 + \left(\frac{1}{\omega c}\right)^2$$

$$0.25\,R^2 = \left(\frac{1}{\omega c}\right)^2 ;\ 0.5\,R = \frac{1}{500 \times C} ;\ C = \frac{1}{250R} ;\ RC = \frac{1}{250}\text{sec}$$

τ = 4 millisecond; τ = 4

35. A train is moving along a straight line with a constant acceleration 'a'. A boy standing in the train throws a ball forward with a speed of 10 m/s, at an angle of 60° to the horizontal. The boy has to move forward by 1.15 m inside the train to catch the ball back at the initial height. The acceleration of the train, in m/s^2, is

Ans. (5)

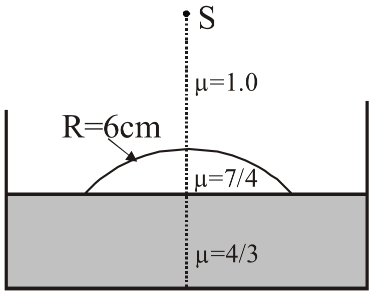
With respect to train :

Velocity : Acceleration :

$$T = \frac{2v_y}{g} = \frac{2 \times 5\sqrt{3}}{10} = \sqrt{3}$$

$$1.15 = 5t - \frac{1}{2}at^2 \Rightarrow a = 5 \text{ m/s}^2$$

36. Water (with refractive index = $\frac{4}{3}$) in a tank is 18 cm deep. Oil of refractive index $\frac{7}{4}$ lies on water making a convex surface of radius of curvature 'R = 6 cm' as shown. Consider oil to act as a thin lens. An object 'S' is placed 24 cm above water surface. The location of its image is at 'x' cm above the bottom of the tank. Then 'x' is



Ans. (2)

First refraction:

$$\mu_1 = 1,\ u = -24,\ \mu_2 = \frac{7}{4},\ R = +6,\ \frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

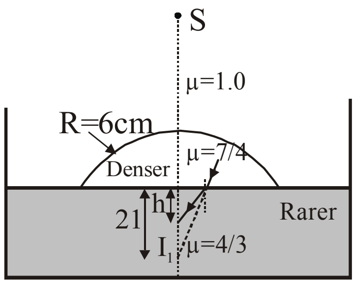
After solving v = 21

Now for second refraction :

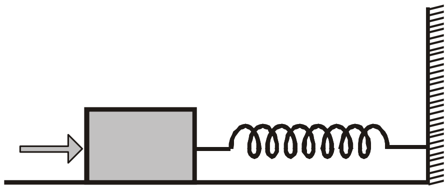
$$h = \frac{21}{(21/16)} = 16$$

So, from bottom 18 – 16 = 2

So, x =2



37. A block of mass 0.18 kg is attached to a spring of force-constant 2 N/m. The coefficient of friction between the block and the floor is 0.1. Initially the block is at rest and the spring is un-stretched. An impulse is given to the block as shown in the figure. The block slides a distance of 0.06 m and comes to rest for the first time. The initial velocity of the block in m/s is V= N/10. Then N is



Ans. (4)

$$-\mu mgx - \frac{1}{2}kx^2 = 0 - \frac{1}{2}mv^2$$

$$v^2 = \frac{1.44}{9} = \frac{4}{10} \Rightarrow \text{N} = 4$$

38. A silver sphere of radius 1 cm and work function 4.7 eV is suspended from an insulating thread in free-space. It is under continuous illumination of 200 nm wavelength light. As photoelectrons are emitted, the sphere gets charged and acquires a potential. The maximum number of photoelectrons emitted from the sphere is A × 10^z (where 1 < A < 10). The value of 'Z' is

Ans. (7)

$$\frac{hc}{\lambda} - \phi = eV = e\frac{(Ne)K}{R}$$

$$\left(\frac{1240}{200} - 4.7\right)1.6 \times 10^{-19} = \frac{N\left(1.6 \times 10^{-19}\right)^2 9 \times 10^9}{1/100}$$

$$\frac{15}{1.6} \times 10^7 = N$$

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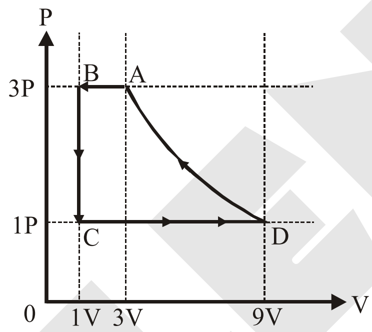
SECTION–IV : (Total Marks : 16)

(Matrix-Match Type)

This Section contains **2 questions**. Each question has **four statements** (A, B, C and D) given in **Column I** and five statements (p, q, r, s and t) in **Column II**. Any given statement in Column I can have correct matching with **ONE** or **MORE** statement(s) given in Column II. For example, if for a given question, statement B matches with the statements given in q and r, then for the particular question, against statement B, darken the bubbles corresponding to q and r in the ORS.

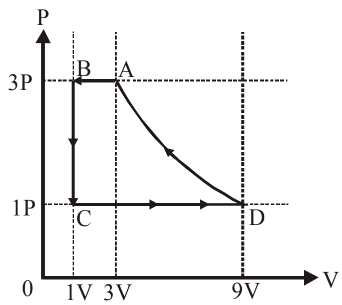
---

39. One mole of a monatomic ideal gas is taken through a cycle ABCDA as shown in P-V diagram. **Column II** gives the characteristics involved in the cycle. Match them with each of the processes given in **Column I**.



Column I			**Column II**
(A)	Process A→B	(p)	Internal energy decreases.
(B)	Process B→C	(q)	Internal energy increases.
(C)	Process C→D	(r)	Heat is lost.
(D)	Process D→A	(s)	Heat is gained.
		(t)	Work is done on the gas.

Ans. **(A) p,r,t (B) p,r (C) q,s (D) r,t**



For (A) : In process AB (isobaric compression)

Work is negative, ΔU is negative , ΔQ is negative

For (B) : BC process (Isochoric)

Work is zero, ΔU is negative , ΔQ is negative

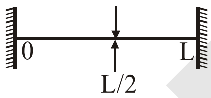
For (C) : CD Process (Isobaric expansion)

Work is negative, ΔU is positive , ΔQ is positive

For (D) : DA Process (V = decreases Isothermal)

Work is negative, ΔU is zero , ΔQ is negative

40. Column I shows four systems, each of the same length L, for producing standing waves. The lowest possible natural frequency of a system is called its fundamental frequency, whose wavelength is denoted as λ_f. Match each system with statements given in **Column II** describing the nature and wavelength of the standing waves.

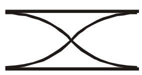
	Column I		**Column II**
(A)	Pipe closed at one end 	(p)	Longitudinal waves
(B)	Pipe open at both ends 	(q)	Transverse Waves
(C)	Stretched wire clamped at both ends 	(r)	λ_f=L
(D)	Stretched wire clamped at both ends and at mid-point 	(s)	$\lambda_f = 2L$
		(t)	$\lambda_f = 4L$

Ans. (A) p,t (B) p,s (C) q,s (D) q,r

For (A) : Sound wave is longitudinal wave

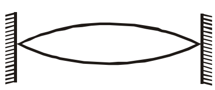
$\frac{\lambda_F}{4} = L \Rightarrow \lambda_F = 4L$

For (B) : Sound wave is longitudinal wave

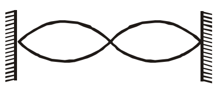


$\frac{\lambda_F}{2} = L \Rightarrow \lambda_F = 2L$

For (C) : String wave is transverse



$\frac{\lambda_F}{2} = L \Rightarrow \lambda_F = 2L$

For (D) :  $\lambda_F = L \Rightarrow$ so (q & r)

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PART - III (MATHEMATICS)

SECTION–I : (Total Marks : 24)
(Single Correct Answer Type)

This section contains **8 multiple choice questions**. Each question has four choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

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41. Let P(6, 3) be a point on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. If the normal at the point P intersects the x-axis at (9,0), then the eccentricity of the hyperbola is -

(A) $\sqrt{\frac{5}{2}}$ (B) $\sqrt{\frac{3}{2}}$ (C) $\sqrt{2}$ (D) $\sqrt{3}$

Sol. Ans. (B)

Equation of normal at P(6, 3) on $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is $\frac{a^2x}{6}+\frac{b^2y}{3}=a^2e^2$

It intersects x-axis at (9, 0)

$\Rightarrow \quad a^2\frac{9}{6}=a^2e^2 \quad \Rightarrow \quad e=\sqrt{\frac{3}{2}}$

42. Let (x,y) be any point on the parabola $y^2 = 4x$. Let P be the point that divides the line segment from (0,0) to (x,y) in the ratio 1 : 3. Then the locus of P is-

(A) $x^2 = y$ (B) $y^2 = 2x$ (C) $y^2 = x$ (D) $x^2 = 2y$

Sol. Ans. (C)

Let P be (h, k)

on using section formula $P\left(\frac{x}{4},\frac{y}{4}\right)$

$\therefore \quad h=\frac{x}{4}$ and $k=\frac{y}{4}$

$\Rightarrow \quad x = 4h$ and $y = 4k$

$\because \quad (x, y)$ lies on $y^2 = 4x$

$\therefore \quad 16k^2 = 16h \Rightarrow \quad k^2 = h$

Locus of point P is $y^2 = x$.

43. Let $f(x) = x^2$ and $g(x) = \sin x$ for all $x\in R$. Then the set of all x satisfying $(f \circ g \circ g \circ f)(x) = (g \circ g \circ f)(x)$, where $(f \circ g)(x) = f(g(x))$, is-

(A) $\pm\sqrt{n\pi}, n\in\{0,1,2,....\}$ (B) $\pm\sqrt{n\pi}, n\in\{1,2,...\}$

(C) $\frac{\pi}{2}+2n\pi, n\in\{....,-2,-1,0,1,2,.....\}$ (D) $2n\pi, n\in\{....,-2,-1,0,1,2,.....\}$

Sol. Ans. (A)

Given $f(x) = x^2$; $g(x) = \sin x$

$fogogof(x) = \sin^2(\sin x^2)$ and $gogof(x) = \sin(\sin x^2)$

given $fogogof(x) = gogof(x) \quad \Rightarrow \quad \sin^2(\sin x^2) = \sin(\sin x^2)$

$\Rightarrow \quad \sin(\sin x^2) = 0$ or 1 (rejected)

$\sin(\sin x^2)=0 \quad \Rightarrow \quad x^2 = n\pi \quad \Rightarrow \quad x=\pm\sqrt{n\pi};\ x\in\{0,1,2,3,.....\}$

44. Let $f:[-1,2] \to [0,\infty)$ be a continuous function such that $f(x) = f(1-x)$ for all $x \in [-1,2]$. Let $R_1 = \int_{-1}^{2} x f(x)dx$, and R_2 be the area of the region bounded by $y = f(x)$, $x=-1$, $x=2$, and the x-axis. Then -

(A) $R_1 = 2R_2$ (B) $R_1 = 3R_2$ (C) $2R_1 = R_2$ (D) $3R_1 = R_2$

Sol. Ans. (C)

$$R_2 = \int_{-1}^{2} f(x)dx, \qquad R_1 = \int_{-1}^{2} x f(x)dx$$

$$= \int_{-1}^{2} (1-x)f(1-x)dx \qquad \left(\because \int_a^b f(x)dx = \int_a^b f(a+b-x)dx\right)$$

$$= \int_{-1}^{2} (1-x)f(x)dx \qquad (\text{given } f(x) = f(1-x))$$

$$= \int_{-1}^{2} f(x)dx - \int_{-1}^{2} x f(x)dx$$

or $R_1 = R_2 - R_1 \Rightarrow 2R_1 = R_2$

45. If $\lim_{x\to 0}\left[1 + x\ell n(1+b^2)\right]^{\frac{1}{x}} = 2b\sin^2\theta, b > 0$ and $\theta \in (-\pi,\pi]$, then the value of θ is-

(A) $\pm\frac{\pi}{4}$ (B) $\pm\frac{\pi}{3}$ (C) $\pm\frac{\pi}{6}$ (D) $\pm\frac{\pi}{2}$

Sol. Ans. (D)

$$\lim_{x\to 0}\left[1 + x\ell n(1+b^2)\right]^{\frac{1}{x}} = e^{\lim_{x\to 0}\frac{x\ell n(1+b^2)}{x}} = 1 + b^2$$

Hence $1 + b^2 = 2b\sin^2\theta$

$\Rightarrow \sin^2\theta = \frac{1}{2}\left(b + \frac{1}{b}\right) \geq 1$

$\therefore \sin^2\theta = 1 \Rightarrow \sin\theta = \pm 1 \Rightarrow \theta = \pm\frac{\pi}{2}$

46. The circle passing through the point (–1,0) and touching the y-axis at (0,2) also passes through the point -

(A) $\left(-\frac{3}{2}, 0\right)$ (B) $\left(-\frac{5}{2}, 2\right)$ (C) $\left(-\frac{3}{2}, \frac{5}{2}\right)$ (D) (–4,0)

Sol. Ans.(D)

Family of circle which touches y-axis at (0,2) is

$x^2 + (y-2)^2 + \lambda x = 0$

Passing through (–1,0)

$\Rightarrow 1 + 4 - \lambda = 0 \Rightarrow \lambda = 5$

$\therefore x^2 + y^2 + 5x - 4y + 4 = 0$

which satisfy the point (–4,0).

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47. Let $\omega \neq 1$ be a cube root of unity and S be the set of all non-singular matrices of the form $\begin{bmatrix} 1 & a & b \\ \omega & 1 & c \\ \omega^2 & \omega & 1 \end{bmatrix}$, where each of a,b and c is either ω or ω^2. Then the number of distinct matrices in the set S is-

(A) 2 (B) 6 (C) 4 (D) 8

Sol. Ans.(A)

$$\begin{vmatrix} 1 & a & b \\ \omega & 1 & c \\ \omega^2 & \omega & 1 \end{vmatrix}$$

$=1 - c\omega - a(\omega - \omega^2 c) = (1 - c\omega) - a\omega(1 - c\omega) = (1 - c\omega)(1 - a\omega)$

for non singular matrix

$$c \neq \frac{1}{\omega} \ \& \ a \neq \frac{1}{\omega}$$

$\Rightarrow$ $c \neq \omega^2, \ a \neq \omega^2$

$\Rightarrow$ a & c must be ω & b can be ω or ω^2

$\therefore$ total matrices = 2

48. A value of b for which the equations

$$x^2 + bx - 1 = 0$$
$$x^2 + x + b = 0,$$

have one root in common is -

(A) $-\sqrt{2}$ (B) $-i\sqrt{3}$ (C) $i\sqrt{5}$ (D) $\sqrt{2}$

Sol. Ans.(B)

$$\frac{x^2}{b^2+1} = \frac{-x}{b+1} = \frac{1}{1-b}$$

$\Rightarrow$ $x = \frac{b+1}{b-1}$ ...(i)

& $x^2 = \frac{b^2+1}{1-b}$ ...(ii)

from (i) & (ii)

$$\left(\frac{b+1}{b-1}\right)^2 = \frac{b^2+1}{1-b}$$

$\Rightarrow$ $(b^2 + 1)(1 - b) = (b + 1)^2$ $\Rightarrow$ $-b^3 + 1 + b^2 - b = b^2 + 1 + 2b$

$\Rightarrow$ $-b^3 - 3b = 0$ $\Rightarrow$ $b(b^2 + 3) = 0$

$\Rightarrow$ $b = 0, \quad b = \pm\sqrt{3}i$

SECTION–II : (Total Marks : 16)

(Multiple Correct Answer Type)

This section contains **4 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE** may be correct.

---

49. If $f(x)=\begin{cases} -x-\frac{\pi}{2}, & x\le -\frac{\pi}{2} \\ -\cos x, & -\frac{\pi}{2}<x\le 0 \\ x-1, & 0<x\le 1 \\ \ell nx, & x>1 \end{cases}$ then -

(A) $f(x)$ is continuous at $x=-\frac{\pi}{2}$

(B) $f(x)$ is not differentiable at x = 0

(C) $f(x)$ is differentiable at x = 1

(D) $f(x)$ is differentiable at $x=-\frac{3}{2}$

Sol. Ans.(A,B,C,D)

$f\left(-\frac{\pi}{2}^{-}\right)=0$, $f\left(-\frac{\pi}{2}^{+}\right)=0$

$$f'(x)=\begin{cases} -1 & x\le \frac{\pi}{2} \\ \sin x & -\frac{\pi}{2}<x\le 0 \\ 1 & 0<x\le 1 \\ \frac{1}{x} & x>1 \end{cases}$$

$f'(0^-) = 0$, $f'(0^+) = 1$ $\therefore$ not differentiable at x = 0

$f'(1^-) = 1$, $f'(1^+) = 1$ $\therefore$ differentiable at x = 1

as $-\frac{3}{2}\in\left(-\frac{\pi}{2},0\right)$

$f'(x) = \sin x$ which is differentiable at $x=-\frac{3}{2}$

50. Let L be a normal to the parabola $y^2 = 4x$. If L passes through the point (9,6), then L is given by-

(A) $y - x + 3 = 0$ (B) $y + 3x - 33 = 0$ (C) $y + x - 15 = 0$ (D) $y - 2x + 12 = 0$

Sol. Ans. (A,B,D)

Equation of normal is $y = mx - 2m - m^3$

It passes through the point (9, 6) then

$6 = 9m - 2m - m^3$

$\Rightarrow m^3 - 7m + 6 = 0 \Rightarrow (m-1)(m-2)(m+3) = 0 \Rightarrow m = 1, 2, -3$

Equations of normals are $y - x + 3 = 0$, $y + 3x - 33 = 0$ & $y - 2x + 12 = 0$

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51. Let E and F be two independent events. The probability that exactly one of them occurs is $\frac{11}{25}$ and the probability of none of them occurring is $\frac{2}{25}$. If P(T) denotes the probability of occurrence of the event T, then -

(A) $P(E)=\frac{4}{5},\ P(F)=\frac{3}{5}$

(B) $P(E)=\frac{1}{5}, P(F)=\frac{2}{5}$

(C) $P(E)=\frac{2}{5}, P(F)=\frac{1}{5}$

(D) $P(E)=\frac{3}{5}, P(F)=\frac{4}{5}$

Sol. Ans. (A,D)

Let P(E) = x & P(F) = y

According to given condition

$x(1-y)+y(1-x)=\frac{11}{25}$

$\Rightarrow\ x+y-2xy=\frac{11}{25}$ ....(i)

Also, $(1-x)(1-y)=\frac{2}{5}$

$\Rightarrow\ x+y-xy=\frac{23}{25}$ ....(ii)

from (i) & (ii)

$xy=\frac{12}{25},\ x+y=\frac{7}{5}$

Solving this $x=\frac{4}{5}, y=\frac{3}{5}$ or $x=\frac{3}{5},\ y=\frac{4}{5}$

52. Let $f:(0,1)\to R$ be defined by $f(x)=\frac{b-x}{1-bx}$,

where b is a constant such that 0 < b < 1. Then

(A) f is not invertible on (0,1)

(B) $f\neq f^{-1}$ on (0,1) and $f'(b)=\frac{1}{f'(0)}$

(C) $f=f^{-1}$ on (0,1) and $f'(b)=\frac{1}{f'(0)}$

(D) f^{-1} is differentiable on (0,1)

Sol. Ans. (A)

$f:(0,1)\to R$

$f(x)=\frac{b-x}{1-bx}$ $\quad b\in(0,1)$

$\Rightarrow\ f'(x)=\frac{b^2-1}{(bx-1)^2}$

$\Rightarrow\ f'(x)<0\ \forall\ x\in(0,1)$

hence $f(x)$ is decreasing function

hence its range (–1, b)

$\Rightarrow$ co-domain $\neq$ range

$\Rightarrow$ $f(x)$ is non-invertible function

SECTION–III : (Total Marks : 24)

(Integer Answer Type)

This Section contains **6 questions**. The answer to each of the questions is a **single-digit integer,** ranging from 0 to 9. The bubble corresponding to the correct answer is to be darkened in the ORS.

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53. Let $y'(x) + y(x)g'(x) = g(x)g'(x)$, $y(0) = 0$, $x \in R$, where $f'(x)$ denotes $\frac{df(x)}{dx}$ and $g(x)$ is a given non-constant differentiable function on R with $g(0) = g(2) = 0$. Then the value of $y(2)$ is

Sol. Ans. 0

Given $y(0) = 0$, $g(0) = g(2) = 0$

Let $y'(x) + y(x) . g'(x) = g(x) g'(x) \Rightarrow y'(x) + (y(x) - g(x)) g'(x) = 0$

$\Rightarrow \frac{y'(x)}{g'(x)} + y(x) = g(x) \quad \Rightarrow \frac{dy(x)}{dg(x)} + y(x) = g(x)$

$\Rightarrow$ I.F. $= e^{\int d(g(x))} = e^{g(x)} \quad \Rightarrow y(x).e^{g(x)} = \int e^{g(x)} g(x).dg(x)$

$y(x).e^{g(x)} = g(x).e^{g(x)} - e^{g(x)} + c$

put $x = 0$

$\Rightarrow 0 = 0 - 1 + c \quad \Rightarrow c = 1$

$\Rightarrow y(2) . e^{g(2)} = g(2)e^{g(2)} - e^{g(2)} + 1$

$\Rightarrow y(2) = 0 - e^0 + 1 \quad \Rightarrow y(2) = 0$

54. Let $\vec{a} = -\hat{i} - \hat{k}$, $\vec{b} = -\hat{i} + \hat{j}$ and $\vec{c} = \hat{i} + 2\hat{j} + 3\hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{r}.\vec{a} = 0$, then the value of $\vec{r}.\vec{b}$ is

Sol. Ans. 9

$\vec{a} = -\hat{i} - \hat{k}$

$\vec{b} = -\hat{i} + \hat{j}$

$\vec{c} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{r} \times \vec{b} = \vec{c} \times \vec{b}$

Taking cross product by $\vec{a}$

$(\vec{r} \times \vec{b}) \times \vec{a} = (\vec{c} \times \vec{b}) \times \vec{a}$

$\Rightarrow (\vec{r}.\vec{a})\vec{b} - (\vec{b}.\vec{a})\vec{r} = (\vec{c}.\vec{a})\vec{b}) - (\vec{b}.\vec{a})\vec{c} \quad \Rightarrow 0 - \vec{r} = (-1-3)(-\hat{i} + \hat{j}) - (1)(\hat{i} + 2\hat{j} + 3\hat{k})$

$\vec{r} = -3\hat{i} + 6\hat{j} + 3\hat{k}$

$\vec{r}.\vec{b} = 3 + 6 = 9$

55. Let $\omega = e^{i\pi/3}$, and a, b, c, x, y, z be non-zero complex numbers such that

$$a + b + c = x$$
$$a + b\omega + c\omega^2 = y$$
$$a + b\omega^2 + c\omega = z.$$

Then the value of $\frac{|x|^2 + |y|^2 + |z|^2}{|a|^2 + |b|^2 + |c|^2}$ is

Sol. Ans. 3

Comment : If $\omega = e^{i\pi/3}$ then $\frac{|x|^2 + |y|^2 + |z|^2}{|a|^2 + |b|^2 + |c|^2}$ is not always an integer, infect its value depends upon a,b,c

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$\Rightarrow$ Let $\omega = e^{i\,2\pi/3}$

$|x|^2 = (a+b+c)\left(\bar{a}+\bar{b}+\bar{c}\right)$

$= |a|^2+|b|^2+|c|^2+a\bar{b}+a\bar{c}+b\bar{a}+b\bar{c}+c\bar{a}+c\bar{b}$

$|y|^2 = (a+b\omega+c\omega^2)\left(\bar{a}+\bar{b}\omega^2+\bar{c}\omega\right)$

$= |a|^2+|b|^2+|c|^2+a\bar{b}\omega^2+a\bar{c}\omega+b\bar{a}\omega+b\bar{c}\omega^2+c\bar{a}\omega^2+c\bar{b}\omega$

$|z|^2=(a+b\omega^2+c\omega)(\bar{a}+\bar{b}\omega+\bar{c}\omega^2)$

$= |a|^2+|b|^2+|c^2|+a\bar{b}\omega+a\bar{c}\omega^2+b\bar{a}\omega^2+b\bar{c}\omega+c\bar{a}\omega+c\bar{b}\omega^2$

$\therefore\ |x|^2+|y|^2+|z|^2 = 3\,(|a|^2+|b|^2+|c|^2)$

$$\Rightarrow\ \frac{|x|^2+|y|^2+|z|^2}{|a|^2+|b|^2+|c|^2}=3$$

56. Let M be 3×3 matrix satisfying

$$M\begin{bmatrix}0\\1\\0\end{bmatrix}=\begin{bmatrix}-1\\2\\3\end{bmatrix},\ M\begin{bmatrix}1\\-1\\0\end{bmatrix}=\begin{bmatrix}1\\1\\-1\end{bmatrix}\text{ and } M=\begin{bmatrix}1\\1\\1\end{bmatrix}=\begin{bmatrix}0\\0\\12\end{bmatrix}$$

Then the sum of the diagonal entries of M is

Sol. Ans. 9

$$\text{Let } M=\begin{bmatrix}a & b & c\\ x & y & z\\ \ell & m & n\end{bmatrix}$$

according to question

$$\begin{bmatrix}a & b & c\\ x & y & z\\ \ell & m & n\end{bmatrix}\begin{bmatrix}0\\1\\0\end{bmatrix}=\begin{bmatrix}-1\\2\\3\end{bmatrix}$$

$\Rightarrow$ b =–1, y =2, m = 3 ...(1)

$$\Rightarrow\ \begin{bmatrix}a & b & c\\ x & y & z\\ \ell & m & n\end{bmatrix}\begin{bmatrix}1\\-1\\0\end{bmatrix}=\begin{bmatrix}1\\1\\-1\end{bmatrix}$$

a – b = 1
x – y = 1
ℓ – m = –1

from (1) a = 0
x = 3
ℓ = 2

$$\begin{bmatrix}a & b & c\\ x & y & z\\ \ell & m & n\end{bmatrix}\begin{bmatrix}1\\1\\1\end{bmatrix}=\begin{bmatrix}0\\0\\12\end{bmatrix}$$

$\ell + m + n = 12$

$\Rightarrow \quad 2 + 3 + n = 12 \quad \Rightarrow \quad n = 7$

Now $a + y + n = 0 + 2 + 7 = 9$

57. The number of distinct real roots of $x^4 - 4x^3 + 12x^2 + x - 1 = 0$ is

Sol. Ans. 2

Let $f(x) = x^4 - 4x^3 + 12x^2 + x - 1$

$f'(x) = 4x^3 - 12x^2 + 24x$

$f''(x) = 12x^2 - 24x + 24$

$= 12(x^2 - 2x + 2) > 0$

$\Rightarrow$ $f'(x)$ is strictly increasing function

$\because$ $f'(x)$ is cubic polynomial

hence number of roots of $f'(x) = 0$ is 1

$\Rightarrow$ Number of maximum roots of $f(x) = 0$ are 2

Now $f(0) = -1,\ f(1) = 9,\ f(-1) = 15$

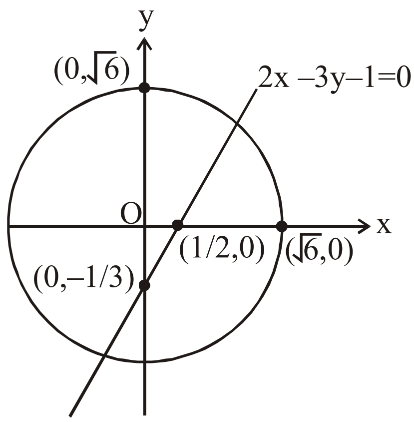
$\Rightarrow$ $f(x)$ has exactly 2 distinct real roots.

58. The straight line $2x - 3y = 1$ divides the circular region $x^2 + y^2 \le 6$ into two parts. If

$$S = \left\{\left(2, \frac{3}{4}\right), \left(\frac{5}{2}, \frac{3}{4}\right), \left(\frac{1}{4}, -\frac{1}{4}\right), \left(\frac{1}{8}, \frac{1}{4}\right)\right\},$$

then the number of point(s) in S lying inside the smaller part is

Sol. Ans. 2



If the point lies inside the smaller part, then origin and point should give opposite signs w.r.t. line & point should lie inside the circle.

for origin : $2 \times 0 - 3 \times 0 - 1 = -1$ (–ve)

for $(2, \frac{3}{4})$: $2\times2-3\times\frac{3}{4}-1=\frac{3}{4}$ (+ve); point lies inside the circle

for $(\frac{5}{2}, \frac{3}{4})$: $2\times\frac{5}{2}-3\times\frac{3}{4}-1=\frac{7}{4}$ (+ve) ; point lies outside the circle

For $\left(\frac{1}{4}, -\frac{1}{4}\right)$: $2\times\frac{1}{4}-3\left(-\frac{1}{4}\right)-1=\frac{1}{4}$ (+ve) ; point lies inside the circle

For $\left(\frac{1}{8}, \frac{1}{4}\right)$: $2\times\frac{1}{8}-3\left(\frac{1}{4}\right)-1=\frac{-3}{2}$ (–ve) ; point lies inside the circle.

$\therefore$ 2 points lie inside smaller part.

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SECTION–IV : (Total Marks : 16)

(Matrix-Match Type)

This Section contains **2 questions**. Each question has **four statements** (A, B, C and D) given in **Column I** and five statements (p, q, r, s and t) in **Column II**. Any given statement in Column I can have correct matching with **ONE** or **MORE** statement(s) given in Column II. For example, if for a given question, statement B matches with the statements given in q and r, then for the particular question, against statement B, darken the bubbles corresponding to q and r in the ORS.

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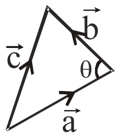
59. Match the statements given in **Column I** with the values given in **Column II**

Column I		**Column II**
(A) If $\vec{a} = \hat{j} + \sqrt{3}\hat{k}$, $\vec{b} = -\hat{j} + \sqrt{3}\hat{k}$ and $\vec{c} = 2\sqrt{3}\hat{k}$ form a triangle, then the internal angle of the triangle between $\vec{a}$ and $\vec{b}$ is	(p)	$\frac{\pi}{6}$
(B) If $\int_a^b (f(x) - 3x)dx = a^2 - b^2$, then the value of $f\left(\frac{\pi}{6}\right)$ is	(q)	$\frac{2\pi}{3}$
(C) The value of $\frac{\pi^2}{\ell n 3}\int_{7/6}^{5/6} \sec(\pi x)dx$ is	(r)	$\frac{\pi}{3}$
(D) The maximum value of $\left|\text{Arg}\left(\frac{1}{1-z}\right)\right|$ for	(s)	π
$\|z\| = 1$, $z \neq 1$ is given by	(t)	$\frac{\pi}{2}$

Sol. Ans. (A) → (q); (B) → (p); (C) → (s); (D) → (t)

(A) 

$|\vec{a}| = 2$
$|\vec{b}| = 2$
$|\vec{c}| = 2\sqrt{3}$

$$\cos\theta = \frac{|\vec{a}|^2 + |\vec{b}|^2 - |\vec{c}|^2}{2|\vec{a}||\vec{b}|} = \frac{4+4-12}{2.2.2} = \frac{-1}{2} \quad \Rightarrow \quad \theta = \frac{2\pi}{3}$$

(B) $\int_a^b (f(x) - 3x)dx = a^2 - b^2 = \int_a^b (-2x)dx$

$\Rightarrow \int_a^b (f(x) - x)dx = 0 \Rightarrow$ one of the possible solution of this equation is

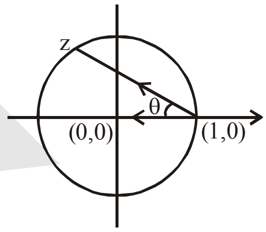
$f(x) = x \quad \Rightarrow \quad f\left(\frac{\pi}{6}\right) = \frac{\pi}{6}$

(C) $\dfrac{\pi^2}{\ell n 3}\displaystyle\int_{7/6}^{5/6}(\sec \pi x)dx = \dfrac{\pi^2}{\ell n 3}\dfrac{1}{\pi}[\,\ell n\,|\sec \pi x + \tan \pi x\,|]_{7/6}^{5/6}$

$$= \frac{\pi}{\ell n 3}\ell n\left|\frac{\sec\frac{5\pi}{6}+\tan\frac{5\pi}{6}}{\sec\frac{7\pi}{6}+\tan\frac{7\pi}{6}}\right| = \frac{\pi}{\ell n 3}\ell n\ 3 = \pi$$

(D) Let $\theta = \text{Arg}\left(\dfrac{1}{1-z}\right)$

$\Rightarrow$ $\theta = \text{Arg}\left(\dfrac{0-1}{z-1}\right)$ which is shown in adjacent diagram.



$\Rightarrow$ Maximum value of θ is approaching to $\dfrac{\pi}{2}$

60. Match the statements given in **Column I** with the intervals/union of intervals given in **Column II**

Column I		Column II
(A) The set $\left\{\text{Re}\left(\dfrac{2iz}{1-z^2}\right) : z \text{ is a complex number, } \lvert z\rvert=1,\ z \neq \pm 1\right\}$ is	(p)	$(-\infty,-1)\cup(1,\infty)$
(B) The domain of the function $f(x) = \sin^{-1}\left(\dfrac{8(3)^{x-2}}{1-3^{2(x-1)}}\right)$ is	(q)	$(-\infty,0)\cup(0,\infty)$
(C) If $f(\theta) = \begin{vmatrix} 1 & \tan\theta & 1 \\ -\tan\theta & 1 & \tan\theta \\ -1 & -\tan\theta & 1 \end{vmatrix}$, then the set	(r)	$[2,\infty)$
$\left\{f(\theta) : 0 \le \theta < \dfrac{\pi}{2}\right\}$ is	(s)	$(-\infty,-1]\cup[1,\infty)$
(D) If $f(x) = x^{3/2}(3x-10),\ x \ge 0$, then $f(x)$ is increasing in	(t)	$(-\infty,0]\cup[2,\infty)$

Sol. Ans. (A) → (s); (B) → (t); (C) → (r); (D) → (r)

(A) Let $z = \cos\theta + i\sin\theta$

$$\text{Re}\left(\frac{2i(\cos\theta + i\sin\theta)}{1-(\cos\theta + i\sin\theta)^2}\right) = \text{Re}\left(\frac{\cos\theta i - \sin\theta}{\sin^2\theta - i\cos\theta\sin\theta}\right)$$

$$= \text{Re}\left(-\frac{1}{\sin\theta}\right) = \frac{-1}{\sin\theta}$$

$\therefore$ Set will be $(-\infty,\ -1]\cup[1,\ \infty)$

(B) $-1 \le \dfrac{8.3^{(x-2)}}{1-3^{2(x-1)}} \le 1 \qquad x \neq 1$

$\Rightarrow \quad -1 \le \dfrac{8.3^{x}}{(3-3^{x})(3+3^{x})} \le 1$

$3^x = t \quad \therefore \quad t > 0$

$\dfrac{8t}{(3-t)(t+3)} \ge -1 \;\Rightarrow\; t \in (0,3) \cup [9, \infty) \;\Rightarrow\; x \in (-\infty, 1) \cup [2, \infty)$

$\dfrac{8t}{(3-t)(t+3)} \le 1 \;\Rightarrow\; t \in (0, 1] \cup (3, \infty) \;\Rightarrow\; x \in (-\infty, 0] \cup (1, \infty)$

Taking intersection,

$x \in (-\infty, 0] \cup [2, \infty)$

(C) $f(\theta) = \begin{vmatrix} 1 & \tan & 1 \\ -\tan\theta & 1 & \tan\theta \\ -1 & -\tan\theta & 1 \end{vmatrix}$

$C_1 \to C_1 + C_3$

$\Rightarrow \quad f(\theta) = \begin{vmatrix} 2 & \tan\theta & 1 \\ 0 & 1 & \tan\theta \\ 0 & -\tan\theta & 1 \end{vmatrix}$

$\Rightarrow \quad f(\theta) = 2\sec^2\theta \qquad \Rightarrow \quad f(\theta) \in [2, \infty)$

(D) $f(x) = 3x^{5/2} - 10x^{3/2}$

$f'(x) = \dfrac{15}{2}x^{3/2} - \dfrac{30}{2}x^{1/2} > 0$

$\Rightarrow \quad \dfrac{15}{2}\sqrt{x}\,(x-2) \ge 0 \quad \Rightarrow \quad x \ge 2$